Integrated Navigation System Using Sigma-Point Kalman Filter and Particle Filter

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ABSTRACT

The paper describes integration of Inertial Navigation System (INS) and Global Positioning System (GPS) navigation systems using new approaches for navigation information processing based on efficient Sigma-Point Kalman filtering and Particle filtering. The paper points out the inherent shortcomings in using the linearization techniques in standard Kalman filters (like Linearized Kalman filter or Extended Kalman filter) and presents, as an alternative, a family of improved derivativeless nonlinear filters. The integrated system was created in a simulation environment. An original contribution of the work consists in creation of models in the simulation environment to confirm the algorithms. The work results are represented in a chart and supported by statistical data to confirm the rightness of the algorithms developed.

1.0 INTRODUCTION

Accurate and reliable navigation systems will have an important role for enhanced military capabilities in the coming years. The INS and GPS are widely used navigation systems in several applications. The main reason for their usage is their dimensions and weight, and their relatively simple implementation in the navigation system. Integrated navigation means that the outputs from two or more navigation sensors are blended to increase the overall accuracy and reliability of the navigation system. Due to its reliability, autonomy and short-term accuracy, inertial navigation is usually regarded as the primary source of navigation data. The major drawback of inertial navigation is that initialization and sensor errors cause the computed quantities to drift. To stabilize the drift and ensure long-term accuracy, the inertial navigation system is integrated with one or more aiding sources. Nowadays, the GPS is the standard aiding source. Although, satellite navigation has a widespread use, problems with the GPS such as reception limitation and interference increase the relevance of other aiding navigation sensors.

The main objective of the INS/GPS integration is to merge information from INS and GPS sensors and provide estimates of the states of the vehicle with greater accuracy than relying on the information from the individual sensors. For many years loose and tightly coupled schemes have been used to provide robust solution. These solutions were used in many applications as in automotive, aerospace robotics and other systems where there are needs for precise navigation.

The inertial navigation is based on measurements of vehicle specific forces and rotation rates obtained from on-board instrumentation consisting of triads of gyros and accelerometers that create an IMU (Inertial Measurement Unit). The measurements from the IMU are used for determination of the vehicle position, velocity and attitude using Newton's equations of motion in the navigation computer. The velocity and the position vectors are computed by double integration of the sum of the gravitational and

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the nongravitational accelerations from the accelerometers and the orientation in space is determined by integrating the rotation rates obtained from the three gyros.

The INS may be mechanized in either gimbaled or strapdown configurations. Gimbaled system is usually heavier and more expensive than a strapdown system and that is the reason why a strapdown INS is used for UAVs or other systems where the weight, size and cost play a significant role. Though INS is autonomous and provides good short-term accuracy, its usage as a stand-alone navigation system is limited due to the time-dependent growth of the inertial sensor errors that is the main disadvantage of using the INS. The accuracy of the INS is therefore highly dependent on the sensor quality, navigation system mechanization and dynamics of the flight vehicle.

The GPS is a space based radio navigation system. This system can provide high accuracy positioning anytime and anywhere in the world. The main disadvantage of the GPS system is that the system is not self-contained and autonomous. Accuracy of the GPS system depends on many factors, for instance receiver clock bias, bias due to receiver clock drift, bias due to system clock error, ionospheric delay, tropospheric delay, random noise, etc. However, compared to the INS system, the GPS receiver is low frequency sensor with bounded errors, thus providing the state information at low update rates with non-increasing errors with time.

Inertial Navigation System

- ✓ *High position and velocity accuracy over short term*
- × *Accuracy decreasing with time*
- × *Affected by gravity*
- ✓ *High measurement output rate*
- ✓ *Autonomous*

Global Positioning System

- ✓ *High position and velocity accuracy over long term*
- ✓ *Uniform accuracy, independent of time*
- ✓ *Not sensitive to gravity*
- × *Non-autonomous*
- × *Low measurement output date*

Integrated INS and GPS system

- ✓ *High position and velocity accuracy over long term*
- ✓ *High data rate*
- ✓ *Navigation output during GPS signal outages*
- ✓ *Precise attitude determination*

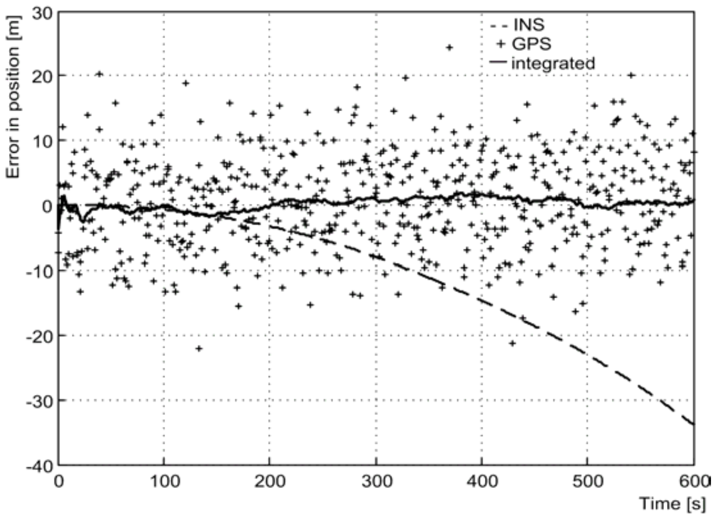

Figure 1: Errors of navigation sensors

2.0 NONLINEAR FILTERING

The nonlinear filtering problem consists of estimating the states of a nonlinear stochastic dynamical system. The class of systems considered is broad and includes bit/attitude estimation, integrated navigation, and radar or sonar surveillance systems. Because most of these systems are nonlinear and/or non-Gaussian, a significant challenge to engineers and scientists is to find efficient methods for on-line, real-time estimation and prediction of the system states and error statistics from sequential observations. In a broad sense, general approaches to optimal nonlinear filtering can be described by a unified way using the recursive Bayesian approach. The central idea of this recursive Bayesian estimation is to determine the probability density function (PDF) of the state vector of the nonlinear systems conditioned on the available measurements. This posterior density function provides the most complete description of the state estimate of the systems. In linear systems with Gaussian process and measurement noises, an optimal closed-form solution is the well-known Kalman filter. In nonlinear systems the optimal exact solution to the recursive Bayesian filtering problem is intractable since it requires infinite dimensional processes. Therefore, approximate nonlinear filters have been proposed. These approximate nonlinear filters can be categorized into five types [13]:

1) *analytical approximations,*
2) *direct numerical approximations,*
3) *sampling-based approaches,*
4) *Gaussian mixture filters,*
5) *simulation-based filters.*

The most widely used approximate nonlinear filters are the Linearized Kalman filter (LKF) and Extended Kalman filter (EKF) that are representative analytical approximate nonlinear filters. The Kalman filter is used as a tool for stochastic estimation from noisy measurements. The Kalman filter is essentially a set of mathematical equations that implement a predictor-corrector type estimator that is optimal in the sense that it minimizes the estimated error covariance, when some presumed conditions are met.

The EKF is similar to the LKF, but with a few differences. The main difference is that the linearization is performed around a trajectory estimated by the filter, not a pre-computed nominal one as in the LKF. Although the EKF maintains the elegant and computationally efficient recursive update form of the KF, it suffers a number of serious limitations. One of these limitations is that the covariance propagation and update are analytically linearized up to the first-order in the Taylor series expansion, and this suggests that the region of stability may be small since nonlinearities in the system dynamics are not fully accounted for. Consequently, these approximations can introduce large errors in the true mean and covariance.

Comparing the Kalman filtering with other methods of nonlinear filtering, the Kalman filter has a number of practical benefits. For example, there is a successful compromise between computational complexity and flexibility, the mean and covariance are linearly transformable, and the mean and covariance estimates can be used to characterize additional features of the distribution, e.g. significant modes [12].

As was mentioned above, the LKF and EKF simply linearize all nonlinear transformations and substitute Jacobian matrices for the linear transformations in the Kalman filter equations, but these procedures are accompanied by some shortcomings: linearized approximation can be extremely poor in cases when error propagation can't be well approximated by a linear function, linearization can be applied only if the Jacobian matrices exist or in some situations calculation of Jacobian matrices is a very difficult and error-prone process.

Based on these reasons different approaches to nonlinear filtering were developed. In this paper the Sigma-point Kalman filter (SPKF) and Particle filters (PF) are described. These filters belong to the simulation-based category of filters and they will be discussed in more detail in the next two sections.

3.0 PARTICLE FILTERING

Numerical methods known as Monte Carlo methods can be described as statistical simulation methods, where statistical simulation is defined as a method that utilizes sequences of random numbers to perform the simulation. Despite the fact that Monte Carlo methods are known for such a long time only nowadays has progress in technique allowed us to apply these methods to complex applications. Monte Carlo methods are now used routinely in many diverse fields from the simulation of complex physical phenomena.

The sequential Monte Carlo approach is known as the bootstrap filtering, the condensation algorithm, and the particle filtering [6]. Particle filters are simulation-based filtering methods where realizations (samples) of the state vector are produced to obtain an empirical approximation of the joint posterior distribution. In fact, particle filters are "tracking" a variable of interest as it evolves over time, typically with a non-Gaussian probability density function. In particle filters the probability density function is calculated using a likelihood function. For this reason multiple copies (particles) of the variable of interest are used, each with a specific weight and the variable of interest is then obtained by the weighted sum of all the particles, in other words, the normalized importance weight and corresponding particles constitute an approximation of the filtering density [15]. The particle filter is recursive (similarly to LKF and EKF) and operates in two phases: prediction and update. That means that after each operation, each particle is modified according to the variable of interest then its weight is recalculated and particles with small weights are rejected (this process is called resampling).

Particle filter implementation can be described by the following algorithm:

1. Initialization: Generate $\boldsymbol{x}_0^{(i)} \sim p(\boldsymbol{x}_0)$, $i=1,....,N$ sample of the state vector is referred to as a particle.

2. Measurement update: Update the weights by the likelihood

$$\boldsymbol{w}_k^{*(i)} = \boldsymbol{w}_{k-1}^{(i)} \cdot p\left(\boldsymbol{y}_k \mid \boldsymbol{x}_k^{(i)}\right) = \boldsymbol{w}_{k-1}^{(i)} \cdot p_{\boldsymbol{v}_t}\left(\boldsymbol{y}_k - h\left(\boldsymbol{x}_k^{(i)}\right)\right) \qquad i=1,....,N$$

Calculate likelihood by

$$p_{v_k}\left(\boldsymbol{y}_k - h\left(\boldsymbol{x}_k^{(i)}\right)\right) = \frac{1}{\left(2\pi\right)^{\dim(y_k)}\sqrt{|\boldsymbol{R}|}} \cdot \exp\left[-\frac{1}{2}\left(\boldsymbol{y}_k - h\left(\boldsymbol{x}_k^{(i)}\right)\right)^T \cdot \boldsymbol{R}^{-1} \cdot \left(\boldsymbol{y}_k - h\left(\boldsymbol{x}_k^{(i)}\right)\right)\right]$$

and normalize to $\boldsymbol{w}_k^{(i)} = \frac{\boldsymbol{w}_k^{*(i)}}{\sum_{i=1}^{N} \boldsymbol{w}_k^{*(i)}}$

3. Resampling: Replicate particles in proportion to their weights [2]. Only resample when the effective number of samples is less than a threshold $N_{threshold}$.

$$N_{eff} = \frac{1}{\sum_{i=1}^{N}\left(\boldsymbol{w}_k^{(i)}\right)^2} < N_{threshold}, \qquad 1 \le N_{eff} \le N$$

where the upper bound is attained when all particles have the same weight, and the lower bound when all probability mass is at one particle. The threshold can be chosen [1] as $N_{threshold} = 2N/3$.

4. Estimation of states & Prediction of particles:

For estimation (approximation) of states MMSE (Minimum Mean Square Error) or MAP (Maximum A Posteriori Estimate) estimators can be used.

Prediction of new particles according to $\boldsymbol{x}_{k+1}^{(i)} \sim p\left(\boldsymbol{x}_{k+1} \mid \boldsymbol{x}_k^{(i)}\right)$

5. Let $k = k+1$ and iterate to item 2).

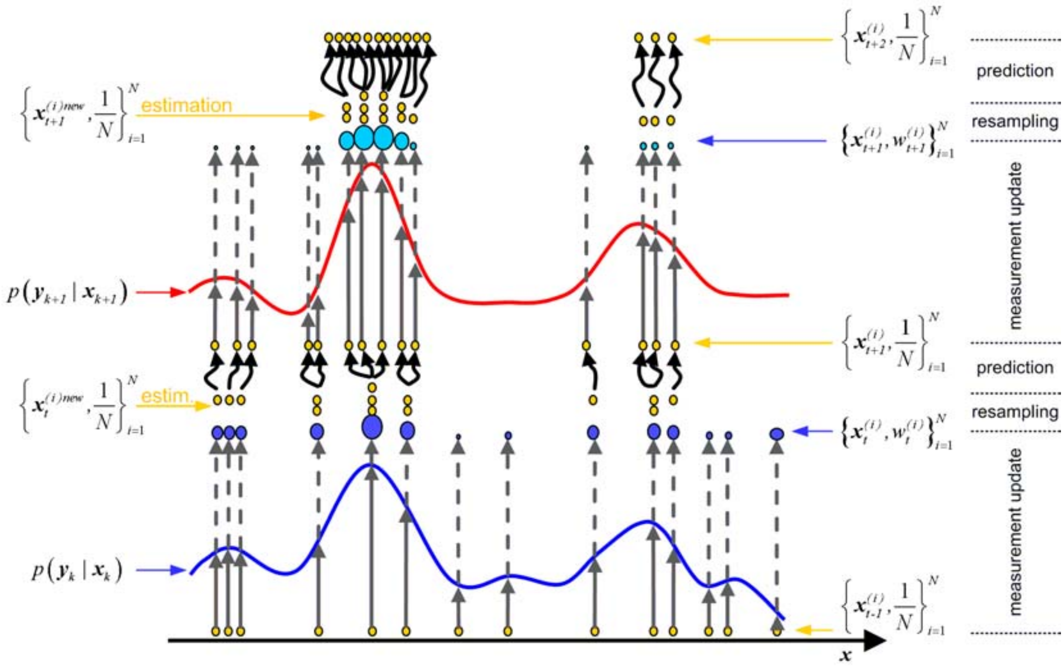

Figure 2: Systematic Diagram for Generic Particle Filtering, similar like in [2]

4.0 SIGMA-POINT KALMAN FILTER

The Sigma-point Kalman filter – also known as an Unscented Kalman filter (UKF) was introduced by Julier and Uhlmann in [11]. In this work they described nonlinear transformation, called the unscented transformation (UT), in which the state probability distribution is represented by a set of sampled sigma points, which are used to parameterize the true mean and covariance of the state distribution.

An unscented transformation is based on two fundamental principles [5]. First, it is easy to perform a nonlinear transformation on a single point. Second, it is not to hard to find a set of individual points in state space whose sample probability density function approximates the true probability density function of the state vector.

The SPKF belongs to a type of sampling-based filters and represents a recursive MMSE estimator, that is a derivative-free (no need for Jacobian and Hessian calculation) alternative to the LKF and EKF. The SPKF is built on the principle that it is easier to approximate a Gaussian distribution than it is to approximate an arbitrary nonlinear function (this is the difference between SPKF and LKF or EKF). In the SPKF, a minimal set of sample points are deterministically chosen (this is the difference between SPKF and PF where the entire probability density function is calculated) and propagated through the original nonlinear system to capture the posterior mean and covariance of a random variable accurately to the 2nd order Taylor series expansion for any nonlinearity.

The Sigma-point Kalman filter can be described by the following algorithm.

1. Initialization:

Set parameters α, β, κ, where α is a constant that determines the spread of the sigma points around the mean of state $\bar{\boldsymbol{x}}$ (usually small positive value $10^{-4} \le \alpha \le 1$), β incorporate prior knowledge of the distribution of $\boldsymbol{x}$ (for Gaussian distribution, $\beta = 2$ is optimal), κ is a secondary scaling parameter (if $\boldsymbol{x}$ is a Gaussian distribution, then $\kappa = 3 - n_x$ is used for multi-dimensional systems).

Initialize:

$$\bar{\boldsymbol{x}}_0 = E\langle \boldsymbol{x}_0 \rangle \qquad \boldsymbol{P}_0 = E\left\langle \langle \boldsymbol{x}_0 - \bar{\boldsymbol{x}}_0 \rangle \langle \boldsymbol{x}_0 - \bar{\boldsymbol{x}}_0 \rangle^T \right\rangle$$

Redefine state vector to new augmented state vector $\boldsymbol{x}^a = \begin{bmatrix} \boldsymbol{x}^T & \boldsymbol{w}^T & \boldsymbol{v}^T \end{bmatrix}^T$.

$$\bar{\boldsymbol{x}}_0^a = E\langle \boldsymbol{x}_0^a \rangle = \begin{bmatrix} \bar{\boldsymbol{x}}_0^T & 0 & 0 \end{bmatrix} \qquad \boldsymbol{P}_0^a = E\left\langle \langle \boldsymbol{x}_0^a - \bar{\boldsymbol{x}}_0^a \rangle \langle \boldsymbol{x}_0^a - \bar{\boldsymbol{x}}_0^a \rangle^T \right\rangle = \begin{bmatrix} \boldsymbol{P}_0 & 0 & 0 \\ 0 & \boldsymbol{Q} & 0 \\ 0 & 0 & \boldsymbol{R} \end{bmatrix}$$

where $\boldsymbol{Q}$ is covariance of process noise, $\boldsymbol{R}$ is covariance of measurement noise.

Calculate composite scaling parameter $\lambda = \alpha^2 (n_a + \kappa) - n_a$ and weights associated with the ith point

$$W^{(0)(mean)} = \lambda / (n_a + \lambda) \qquad W^{(0)(cov)} = \lambda / (n_a + \lambda) + (1 - \alpha^2 + \beta)$$

$$W^{(i)(mean)} = W^{(i)(cov)} = 0.5 / (n_a + \lambda) \qquad i = 1, \dots, 2n_a$$

where n_a is dimension of augmented state vector $n_a = n_x + n_w + n_v$, n_x is dimension of state vector, n_w is dimension of process noise, n_v is dimension of measurement noise.

2. Calculate the sigma points

$$\boldsymbol{\mathcal{X}}_{k-1}^{a} = \left[\left(\boldsymbol{\mathcal{X}}_{k-1}^{x}\right)^{T} \left(\boldsymbol{\mathcal{X}}_{k-1}^{w}\right)^{T} \left(\boldsymbol{\mathcal{X}}_{k-1}^{v}\right)^{T} \right]^{T} = \left[\bar{\boldsymbol{x}}_{k-1}^{a} \quad \bar{\boldsymbol{x}}_{k-1}^{a} + \left(\sqrt{\left(n_a + \kappa\right) \cdot \boldsymbol{P}_{k-1}^{a}}\right) \quad \bar{\boldsymbol{x}}_{k-1}^{a} - \left(\sqrt{\left(n_a + \kappa\right) \cdot \boldsymbol{P}_{k-1}^{a}}\right) \right]$$

3. Time update

$$\boldsymbol{\mathcal{X}}_{k,k-1}^{x} = \boldsymbol{f}\left(\boldsymbol{\mathcal{X}}_{k-1}^{x}, \boldsymbol{\mathcal{X}}_{k-1}^{w}\right)$$

$$\bar{\boldsymbol{x}}_{k,k-1} = \sum_{i=0}^{2n_a} W^{(mean),(i)} \cdot \boldsymbol{\mathcal{X}}_{k,k-1}^{x,(i)}$$

$$\boldsymbol{P}_{k,k-1} = \sum_{i=0}^{2n_a} W^{(cov),(i)} \cdot \left[\boldsymbol{\mathcal{X}}_{k,k-1}^{x,(i)} - \bar{\boldsymbol{x}}_{k,k-1}\right] \cdot \left[\boldsymbol{\mathcal{X}}_{k,k-1}^{x,(i)} - \bar{\boldsymbol{x}}_{k,k-1}\right]^{T}$$

$$\boldsymbol{\mathcal{Y}}_{k,k-1} = \boldsymbol{h}\left(\boldsymbol{\mathcal{X}}_{k,k-1}^{x}, \boldsymbol{\mathcal{X}}_{k-1}^{v}\right)$$

$$\bar{\boldsymbol{y}}_{k,k-1} = \sum_{i=0}^{2n_a} W^{(mean),(i)} \cdot \boldsymbol{\mathcal{Y}}_{k,k-1}^{(i)}$$

4. Measurement update

$$\boldsymbol{P}_{\tilde{\boldsymbol{y}}_k \tilde{\boldsymbol{y}}_k} = \sum_{i=0}^{2n_a} W^{(cov),(i)} \cdot \left[\boldsymbol{\mathcal{Y}}_{k,k-1}^{(i)} - \bar{\boldsymbol{y}}_{k,k-1}\right] \cdot \left[\boldsymbol{\mathcal{Y}}_{k,k-1}^{(i)} - \bar{\boldsymbol{y}}_{k,k-1}\right]^{T}$$

$$\boldsymbol{P}_{\boldsymbol{x}_k \boldsymbol{y}_k} = \sum_{i=0}^{2n_a} W^{(cov),(i)} \cdot \left[\boldsymbol{\mathcal{X}}_{k,k-1}^{(i)} - \bar{\boldsymbol{x}}_{k,k-1}\right] \cdot \left[\boldsymbol{\mathcal{Y}}_{k,k-1}^{(i)} - \bar{\boldsymbol{y}}_{k,k-1}\right]^{T}$$

$$\boldsymbol{K}_k = \boldsymbol{P}_{\boldsymbol{x}_k \boldsymbol{y}_k} \cdot \boldsymbol{P}_{\tilde{\boldsymbol{y}}_k \tilde{\boldsymbol{y}}_k}^{-1}$$

$$\bar{\boldsymbol{x}}_k = \bar{\boldsymbol{x}}_{k,k-1} + \boldsymbol{K}_k\left(\boldsymbol{y}_k - \bar{\boldsymbol{y}}_{k,k-1}\right)$$

$$\boldsymbol{P}_k = \boldsymbol{P}_{k,k-1} - \boldsymbol{K}_k \cdot \boldsymbol{P}_{\tilde{\boldsymbol{y}}_k \tilde{\boldsymbol{y}}_k} \cdot \boldsymbol{K}_k^{T}$$

5. Let $k = k + 1$ and iterate to item 2).

5.0 SIMULATIONS & RESULTS

The tests of our SPKF and PF approach to filter navigation information were performed in a simulation environment. For this, the trajectory of a moving vehicle was generated and models of the sensor errors (IMU, GPS) were developed. Errors of the IMU were modeled as in navigation grade systems where accelerometer errors are bias $0.1\ mm.s^{-2}$ and noise $0.1\ mm.s^{-2}.Hz^{-1/2}$, and gyro errors are modeled with bias $0.01\ deg.h^{-1}$ and noise $0.005\ deg.h^{-1}.Hz^{-1/2}$. For GPS model was considered receiver working on C/A code with position error $\sigma_{\rho} = 15\ meters,\ \sigma_{\Delta\rho} = 0.01\ m.s^{-1},$ for all used satellites. The integration scheme used in the model was tightly coupled with pseudorange and delta pseudorange measurements.

Processed outputs were compared to the etalon model of the movement depicted in Fig. 3. The trajectory is characterized by minimum changes of movement parameters, for example: maximum speed is only $15\ m.s^{-1}$ also accelerations in all directions are small (the movement of a small R/C aircraft). The trajectory of this object was evaluated by model of the INS and GPS sensors, respectively.

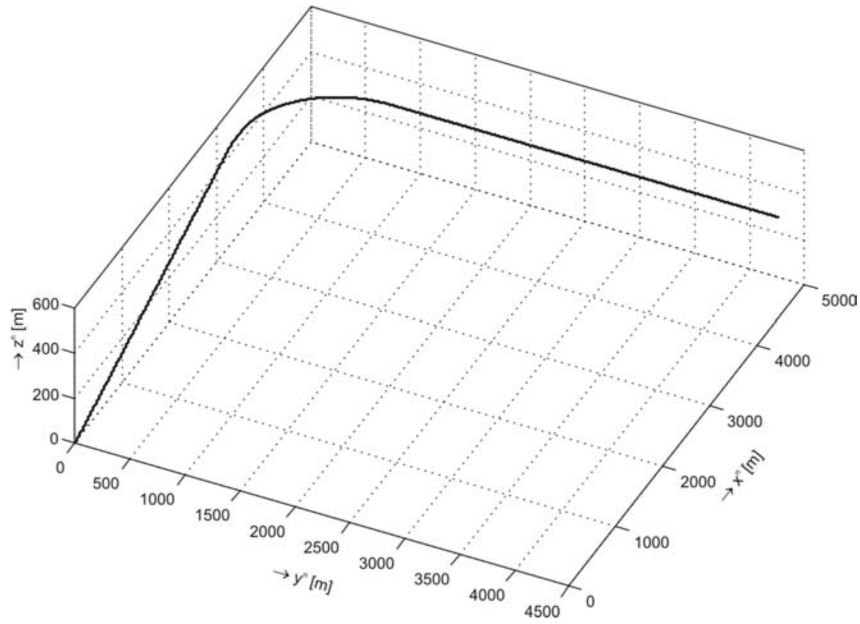

Figure 3. Etalon trajectory of vehicle

The results of EKF, SPKF and PF were compared and evaluated. These results are in table 1, where root-mean-square estimation errors for these three filters are given.

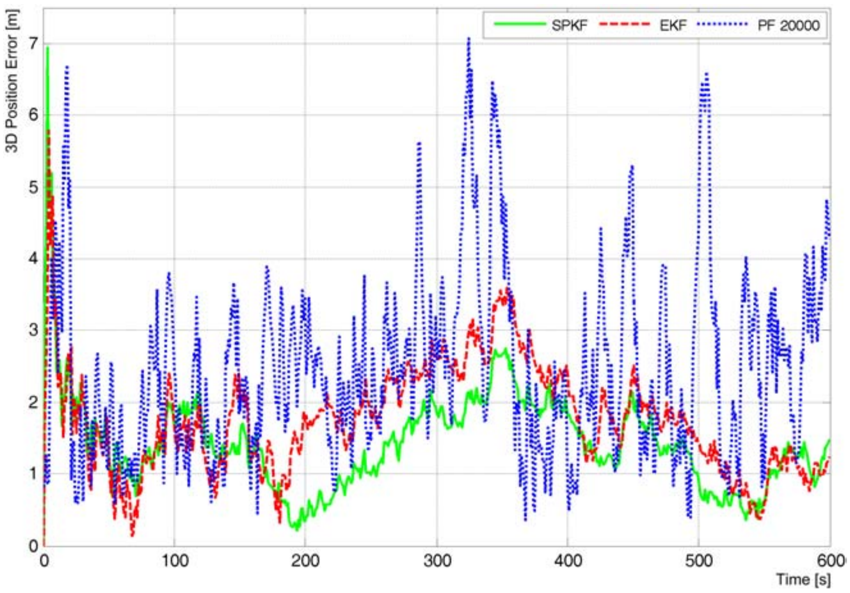

Figure 4: 3D Position error

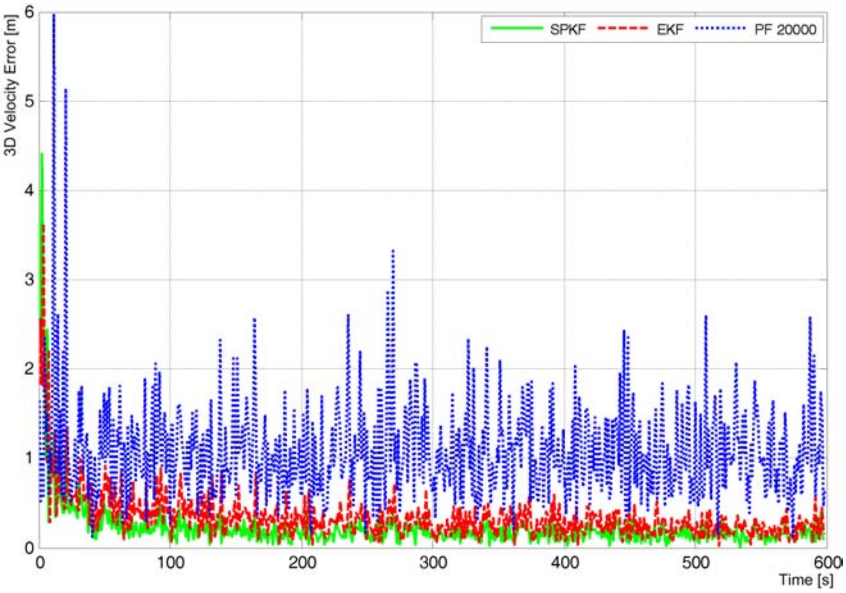

Figure 5: 3D Velocity error

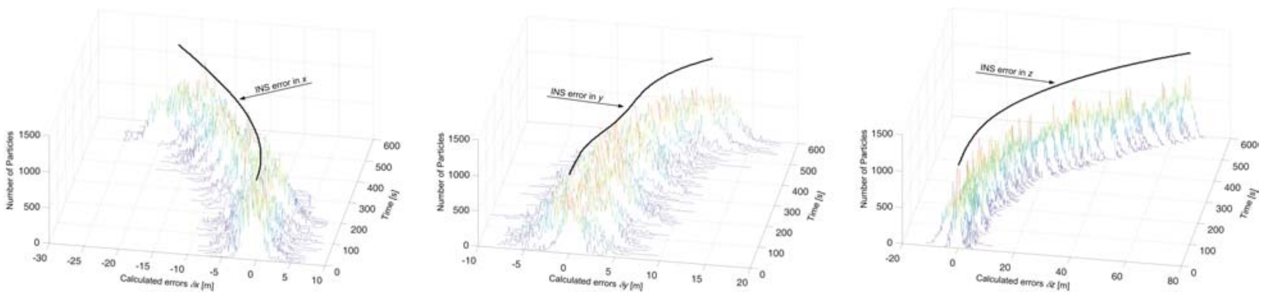

Figure 6: Histogram of particles, and INS error

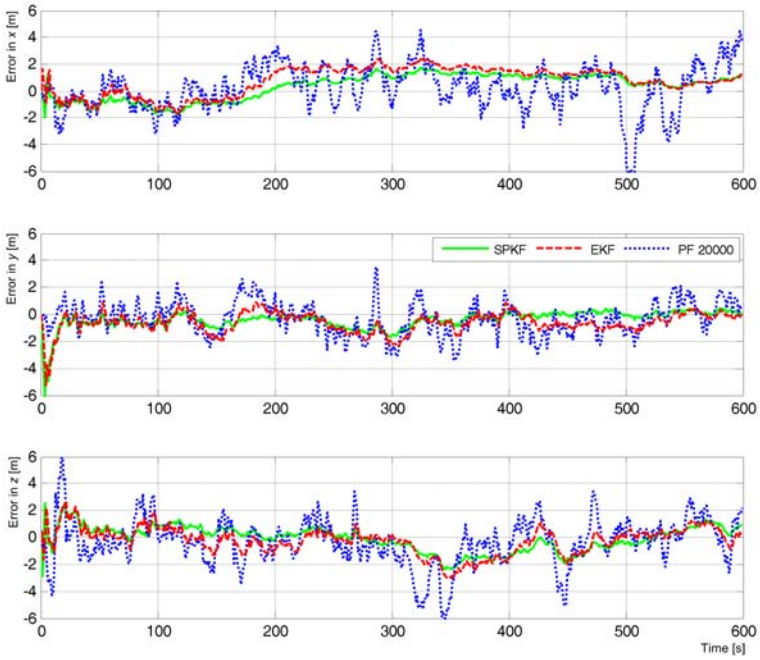

Figure 7: Position errors of the filters during 600s simulation

As results, the statistic data RMS of error (difference between etalon trajectory and trajectory indicated by systems respectively Fig. 4 and Fig. 5) were calculated. The simulation of the particular filtering methods took *600 seconds.* The stand-alone INS maximum error in z^n axis was *-65 meters*, which grows in the time. The calculated maximum GPS position error was approximately *24 meters*.

In figure 7 are depicted trends of filters errors in x^n, y^n and z^n axis, respectively and in figure 4 and figure 5 are depicted trends of 3D position and velocity errors, respectively. In tab. 1 are evaluated statistical parameters of these errors over complete simulated flight trajectory. From this table is unambiguous, that SPKF shows better performance characteristics than EKF and Particle filter.

The Particle filter shows higher error than was expected even we have used 20000 or 50000 particles. The reason is probably in high dimensionality of the model that causes flattening of the approximated probability density function. For better illustration of the particle filter functionality figure 6 was created. There are depicted trends of particles (histograms of all generated and resampled particles) in particular axes for uncoupled integration where is no corrections of INS and GPS systems. The solid lines at the top of each histogram are trends of stand-alone INS errors.

Table 1: Test results

FILTER	3D position error [*m*]	3D velocity error [$m.s^{-1}$]
EKF	0.91	0.35
SPKF	0.68	0.29
PF 20000	1.44	0.65
PF 50000	1.3	0.54

6.0 CONCLUSION & FUTURE WORK

The presented paper describes different approaches to navigation information processing using Particle filter and Sigma-point Kalman filter instead of traditional approaches using Linearized Kalman filter or Extended Kalman filter.

The results show that generic Particle filter is appropriate to use in INS/GPS navigation systems but is not optimal solution in this case. The reason is that the number of particles used in the filter depends on the system dimension. One of the ways how to increase efficiency of PF is using Rao-Blackwellized filter also known as Marginalized particle filter [14, 16]. This filter is split into two parts where on one (nonlinear) part is applied PF and on the second (linear) part is applied Kalman filter. The reason for this is that linear states can be optimally estimated using Kalman filter and we reduce the number of states for particle filtering and the number of particles as well. Main disadvantage of the PF is its computational complexity which corresponds to number of used particles.

Since the Sigma-point Kalman filter accounts for system nonlinearities, from the results it is clear that it is more accurate in state estimation than using standard Kalman filter. Comparing computational complexity, there is no big difference between SPKF and EKF and should be reduced using Reduced Sigma-point Kalman filter that uses simplex sigma points yet [5, 9, 10].

There is also need to say that all tests were conducted under ideal conditions. It means, that we have used ideal placement of the sensors (no lever-arm), the other errors such scale factor, non-orthogonality etc. in the INS were neglected. Also the model of the GPS receiver was simplified. In future we would like make some experimental results with real hardware where algorithms will be tested in real environment to confirm our ideas and also test Sigma-point Particle filter which collects benefits of both presented filters [22].

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